The particle invariance in particle physics

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Abstract

Since particles such as molecules, atoms and nuclei are composite particles, it is important to recognize that physics must be invariant for the composite particles and their constituent particles, this requirement is called particle invariance in this paper. But difficulties arise immediately because for fermion we use the Dirac equation, for meson we use the Klein-Gordon equation and for classical particle we use the Newtonian mechanics, while the connections between these equations are quite indirect. Thus if the particle invariance is hold in physics, i.e., only one physical formalism exists for any particle, we can expect to find out the differences between these equations by employing the particle invariance. As the results, several new relationships between them are found, the most important result is that the obstacles that cluttered the path from classical mechanics to quantum mechanics are found, it becomes possible to derive the Schrodinger equation from the Newtonian mechanics after the obstacles are removed. A improved model is proposed to gain a better understanding on elementary particle interactions. This approach offers enormous advantages, not only for giving the first physically reasonable interpretation of quantum mechanics, but also for improving the quark model.

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1 Introduction

Without doubt, most particles can be regarded as composite particles, such as molecules composed of atoms, atoms composed of electrons and nuclei, nuclei composed of nucleons, so on, it is important to recognize that physics must be invariant for the composite particles and their constituent particles, this requirement is called particle invariance in this paper. But difficulties arise immediately because for fermion we use the Dirac equation, for meson we use the Klein-Gordon equation and for classical particle we use the Newtonian mechanics, while the connections between these equations are quite indirect. Thus if the particle invariance is hold in physics, i.e., only one physical formalism exists for any particle, we can expect to find out the differences between these equations by employing the particle invariance, using this approach is one of the aims of this paper, consequently, several new relationships between them are found, the most important result is that the obstacles that cluttered the path from classical mechanics to quantum mechanics are found, it becomes possible to derive the Schrodinger equation from the Newtonian mechanics after the obstacles are removed.

A quark model of hadrons has been proposed in which mesons are bound states of quarks and antiquarks, and baryons are bound states of three quarks. Detailed calculation based on the quark model of hadrons have been rather successful in accounting for the properties such as magnetic moments and the interactions of hadrons. So far we lack the evidence of the experimental detection of free quarks. Under the particle invariance, our improvement on the quark model starts with introducing a new elementary particle called Dollon to resemble particles such as baryons, mesons or other composite particles, the another aim of this paper is just to discuss the Dollon model, instead of quark model, the Dollon model is better in organizing known data, specially in modelling interactions.

Next aim is just to discuss those interactions between particles based on the Dollon model under the particle invariance, several formulae of interactions are derived and discussed. It is found that the gravitational force and Coulomb's force on a particle always act in the direction orthogonal to the 4-vector velocity of the particle in 4-dimensional space-time, rather than along the line joining a couple of particles, this inference is obviously supported from the fact that the magnitude of the 4-vector velocity is kept constant. Further, the Maxwell's equations can be derived from the classical Coulomb's force and the magnitude formula of 4-vector velocity of particle.

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2 Fermions and Bosons

Fermions satisfy Fermi-Dirac statistics, Bosons satisfy Bose-Einstein statistics, there is a connection between the spin of a particle and the statistics. It is clear that the spin is a key concept for particle physics. In this section we shall show that the spin of a particle is one of consequences of the particle invariance.

According to the Newtonian mechanics, in a hydrogen atom, the single electron revolves in an orbit about the nucleus, its motion can be described with its position in an inertial Cartesian coordinate system S : $(x_1, x_2, x_3, x_4 = ict)$. As the time elapses, the electron draws a spiral path (or orbit), as shown in FIG.1(a) in imagination.

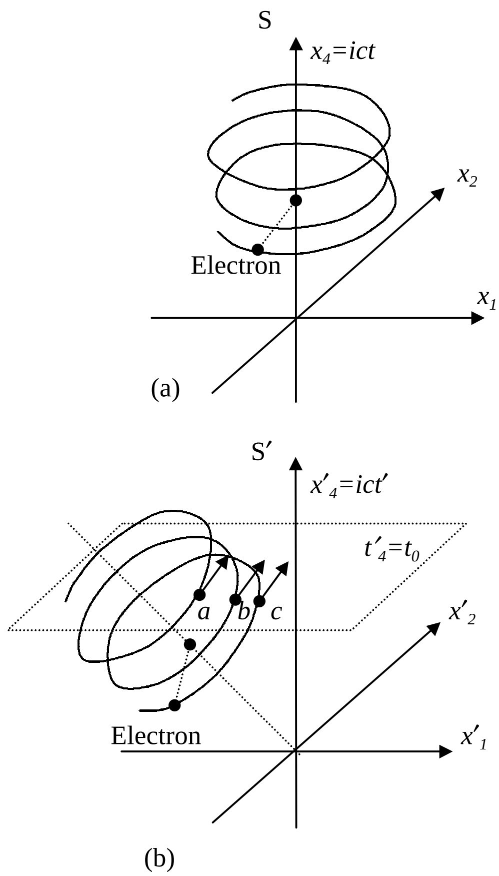

Figure 1: The motion of the electron of hydrogen atom in 4-dimensional space-time.

If the reference frame S rotates through an angle about the x_2-axis in FIG.1(a), becomes a new reference frame S' (there will be a Lorentz transformation linking the frames S and S'), then in the frame S', the spiral path of the electron tilts with respect to the x'_4-axis with the angle as shown in FIG.1(b). At one moment, for example, $t'_4 = t_0$ moment, the spiral path pierces many points at the plane $t'_4 = t_0$, for example, the points labeled a, b and c in FIG.1(b), these points indicate that the electron can appear at many points at the time t_0, in agreement with the concept of the probability in quantum mechanics. This situation gives us a hint to approach quantum wave nature from the Newtonian mechanics.

Because the electron pierces the plane $t'_4 = t_0$ with 4-vector velocity u, at every pierced point we can label a local 4-vector velocity . The pierced points may be numerous if the path winds up itself into a cell about the nucleus (due to a nonlinear effect in a sense), then the 4-vector velocities at the pierced points form a 4-vector velocity field. It is noted that the observation plane selected for the piercing can be taken at an arbitrary orientation, so the 4-vector velocity field may be expressed in general as $u(x_1, x_2, x_3, x_4)$, i.e. the velocity u is of a function of position.

At every point in the reference frame S' the electron satisfies the relativistic Newton's second law

$$m\frac{du_\mu}{d\tau} = qF_{\mu\nu}u_\nu \tag{1}$$

the notations consist with the convention[1]. Since the Cartesian coordinate system is a frame of reference whose axes are orthogonal to one another, there is no distinction between covariant and contravariant components, only subscripts need be used. Here and below, summation over twice repeated indices is implied in all case, Greek indices will take on the values 1,2,3,4, and regarding the mass m as a constant. As mentioned above, the 4-vector velocity u can be regarded as a 4-vector velocity field, then

$$\frac{du_\mu}{d\tau} = \frac{\partial u_\mu}{\partial x_\nu}\frac{dx_\nu}{d\tau} = u_\nu \partial_\nu u_\mu \tag{2}$$

$$qF_{\mu\nu}u_\nu = qu_\nu(\partial_\mu A_\nu - \partial_\nu A_\mu) \tag{3}$$

Substituting them back into Eq.(1), and re-arranging these terms, we obtain

$$\begin{aligned} u_\nu\partial_\nu(mu_\mu + qA_\mu) &= u_\nu\partial_\mu(qA_\nu) \\ &= u_\nu\partial_\mu(mu_\nu + qA_\nu) - u_\nu\partial_\mu(mu_\nu) \\ &= u_\nu\partial_\mu(mu_\nu + qA_\nu) - \frac{1}{2}\partial_\mu(mu_\nu u_\nu) \\ &= u_\nu\partial_\mu(mu_\nu + qA_\nu) - \frac{1}{2}\partial_\mu(-mc^2) \\ &= u_\nu\partial_\mu(mu_\nu + qA_\nu) \end{aligned} \tag{4}$$

Using the notation

$$K_{\mu\nu} = \partial_\mu(mu_\nu + qA_\nu) - \partial_\nu(mu_\mu + qA_\mu) \tag{5}$$

Eq.(4) is given by

$$u_\nu K_{\mu\nu} = 0 \tag{6}$$

Because $K_{\mu\nu}$ contains the variables $\partial_\mu u_\nu$, $\partial_\mu A_\nu$, $\partial_\nu u_\mu$ and $\partial_\nu A_\mu$ which are independent from u_ν, then a solution satisfying Eq.(6) is of

$$\begin{aligned} K_{\mu\nu} &= 0 && (7)\\ \partial_\mu(mu_\nu + qA_\nu) &= \partial_\nu(mu_\mu + qA_\mu) && (8)\end{aligned}$$

The above equation allows us to introduce a potential function Φ in mathematics, further set $\Phi = -i\hbar\ln\psi$, we obtain a very important equation

$$(mu_\mu + qA_\mu)\psi = -i\hbar\partial_\mu\psi \tag{9}$$

where ψ representing the wave nature may be a complex mathematical function, its physical meanings will be determined from experiments after the introduction of the Planck's constant $\hbar$.

Multiplying the two sides of the following magnitude formula of 4-vector velocity of particle by ψ

$$-m^2c^2 = m^2u_\mu u_\mu \tag{10}$$

which is valid at every point in the 4-vector velocity field, and using Eq.(9), we obtain

$$\begin{aligned} -m^2c^2\psi &= mu_\mu(-i\hbar\partial_\mu - qA_\mu)\psi\\ &= (-i\hbar\partial_\mu - qA_\mu)(mu_\mu\psi) - [-i\hbar\psi\partial_\mu(mu_\mu)]\\ &= (-i\hbar\partial_\mu - qA_\mu)(-i\hbar\partial_\mu - qA_\mu)\psi\\ &\quad -[-i\hbar\psi\partial_\mu(mu_\mu)] && (11)\end{aligned}$$

According to the continuity condition for the electron motion

$$\partial_\mu(mu_\mu) = 0 \tag{12}$$

we have

$$-m^2c^2\psi = (-i\hbar\partial_\mu - qA_\mu)(-i\hbar\partial_\mu - qA_\mu)\psi \tag{13}$$

It is known as the Klein-Gordon equation.

On the condition of non-relativity, the Schrodinger equation can be derived from the Klein-Gordon equation [3](P.469).

However, we must admit that we are careless when we use the continuity condition Eq.(12), because, from Eq.(9) we obtain

$$\partial_\mu(mu_\mu) = \partial_\mu(-i\hbar\partial_\mu\ln\psi - qA_\mu) = -i\hbar\partial_\mu\partial_\mu\ln\psi \tag{14}$$

where we have used the Lorentz gauge condition. Thus from Eq.(10) to Eq.(11) we obtain

$$\begin{aligned} -m^2c^2\psi &= (-i\hbar\partial_\mu - qA_\mu)(-i\hbar\partial_\mu - qA_\mu)\psi\\ &\quad +\hbar^2\psi\partial_\mu\partial_\mu\ln\psi && (15)\end{aligned}$$

This is of a complete wave equation for describing the motion of the electron accurately. The Klein-Gordon equation is a linear equation so that the principle of superposition remains valid, however with the addition of the last term of Eq.(15), Eq.(15) turns to display chaos.

In the following we shall show the Dirac equation from Eq.(9) and Eq.(10). From Eq.(9), the wave function can be given in integral form by

$$\Phi = -i\hbar\ln\psi = \int_{x_0}^{x}(mu_\mu + qA_\mu)dx_\mu + \theta \tag{16}$$

where θ is an integral constant, x_0 and x are the initial and final points of the integral with an arbitrary integral path. Since the Maxwell's equations are gauge invariant, Eq.(9) should preserve invariant form under a gauge transformation specified by

$$A'_\mu = A_\mu + \partial_\mu\chi, \quad \psi' \leftarrow \psi \tag{17}$$

where χ is an arbitrary function. Then Eq.(16) under the gauge transformation is given by

$$\begin{aligned} \psi' &= \exp\left\{\frac{i}{\hbar}\int_{x_0}^{x}(mu_\mu + qA_\mu)dx_\mu + \frac{i}{\hbar}\theta\right\}\exp\left\{\frac{i}{\hbar}q\chi\right\}\\ &= \psi\exp\left\{\frac{i}{\hbar}q\chi\right\} && (18)\end{aligned}$$

The situation in which a wave function can be changed in a certain way without leading to any observable effects is precisely what is entailed by a symmetry or invariant principle in quantum mechanics. Here we emphasize that the invariance of velocity field is hold for the gauge transformation.

Suppose there is a family of wave functions $\psi^{(j)}, j = 1,2,3,...,N$, which correspond to the same velocity field denoted by $P_\mu = mu_\mu$, they are distinguishable from their different phase angles θ as in Eq.(16). Then Eq.(10) can be given by

$$0 = P_\mu P_\mu\psi^{(j)}\psi^{(j)} + m^2c^2\psi^{(j)}\psi^{(j)} \tag{19}$$

Suppose there are matrices a_μ which satisfy

$$a_{\nu lj}a_{\mu jk} + a_{\mu lj}a_{\nu jk} = 2\delta_{\mu\nu}\delta_{lk} \tag{20}$$

then Eq.(19) can be rewritten as

$$\begin{aligned} 0 &= a_{\mu kj}a_{\mu jk}P_\mu\psi^{(k)}P_\mu\psi^{(k)}\\ &\quad +(a_{\nu lj}a_{\mu jk} + a_{\mu lj}a_{\nu jk})P_\nu\psi^{(l)}P_\mu\psi^{(k)}|_{\nu\geq\mu,when\nu=\mu,l\neq k}\end{aligned}$$

$$
\begin{aligned}
&+mc\psi^{(j)}mc\psi^{(j)} \\
= \quad & [a_{\nu lj}P_\nu\psi^{(l)} + i\delta_{lj}mc\psi^{(l)}][a_{\mu jk}P_\mu\psi^{(k)} - i\delta_{jk}mc\psi^{(k)}]
\end{aligned} \tag{21}
$$

Where δ_{jk} is the Kronecker delta function, $j,k,l = 1,2,...,N$. For the above equation there is a special solution given by

$$[a_{\mu jk}P_\mu - i\delta_{jk}mc]\psi^{(k)} = 0 \tag{22}$$

There are many solutions for a_μ which satisfy Eq.(20), we select a set of a_μ as

$$N = 4, \quad a_\mu = \gamma_\mu \quad (\mu = 1,2,3,4) \tag{23}$$

$$\gamma_n = -i\beta\alpha_n \quad (n = 1,2,3), \quad \gamma_4 = \beta \tag{24}$$

where γ_μ, α and β are the matrices defined in the Dirac algebra[1](P.557). Substituting them into Eq.(22), we obtain

$$[ic(-i\hbar\partial_4 - qA_4) + c\alpha_n(-i\hbar\partial_n - qA_n) + \beta mc^2]\psi = 0 \tag{25}$$

where ψ is an one-column matrix about $\psi^{(k)}$.

Let index s denote velocity field, then $\psi_s(x)$, whose four component functions correspond to the same velocity field s, may be regarded as the eigenfunction of the velocity field s, it may be different from the eigenfunction of energy. Because the velocity field is an observable in a physical system, in quantum mechanics we know, $\psi_s(x)$ constitute a complete basis in which arbitrary function $\phi(x)$ can be expanded in terms of them

$$\phi(x) = \int C(s)\psi_s(x)ds \tag{26}$$

Obviously, $\phi(x)$ satisfies Eq.(25). Then Eq.(25) is just the Dirac equation.

Alternatively, another method to show the Dirac equation is more traditional: At first, we show the Dirac equation of free particle by employing plane waves, we easily obtain Eq.(25) on the condition of $A_\mu = 0$; Next, adding electromagnetic field, plane waves are valid in any finite small volume with the momentum of Eq.(9) when we regard the field to be uniform in the volume, so the Dirac equation Eq.(25) is valid in the volume even if $A_\mu \neq 0$, plane waves constitute a complete basis in the volume; Third, the finite small volume can be chosen to locate at anywhere, then anywhere have the same complete basis, therefore the Dirac equation Eq.(25) is valid at anywhere.

Of course, on the condition of non-relativity, the Schrodinger equation can be derived from the Dirac equation [3](P.479).

By further calculation, the Dirac equation can arrive at the Klein-Gordon equation with an additional term which represents the effect of spin, this term is just the last term of Eq.(15) approximately.

But, do not forget that the Dirac equation is a special solution of Eq.(21), therefore we believe there are some quantum effects beyond the Dirac equation.

Eq.(21) originates from the magnitude formula of 4-vector velocity of particle, the formula is suitable for any particle, so it satisfies the particle invariance. The Dirac equation is regarded as approximate one to Eq.(21), the approximation brings out many troubles with the spin concept. From the Dirac equation we can predict that a composite particle and an its constituent both have their own spins, but this prediction is not true for mesons because Pion has zero spin while its constituent quark has 1/2 spin, in other words, due to the approximation the Dirac equation does not involve some states such as zero spin state. That is why we want to classify particles into fermions and mesons by spin and use different equations. If we can find a precise solution of Eq.(21) instead of the Dirac equation, then the classification is not necessary. It is noted that Eq.(21) is nonlinear while the Dirac equation is linear, this indicate that we can never find any precise solutions in a linear equation which satisfy Eq.(21). Therefore, for this problem, a good solution depends on how much precision we can reach.

In one hand, it is rather remarkable that the Klein-Gordon equation and the Dirac equation can be derived from the relativistic Newton's second law approximately, in another hand, all particles, such as fermions, bosons and classical particles satisfy the relativistic Newton's second law (it will be further clear later), thus it is a natural choice that only the relativistic Newton's second law is independent and necessary. Only one formalism is necessary for any particle, this is just the particle invariance, we arrive at the aim.

As mentioned above, the spin is one feature hidden in the relativistic Newton's second law, but more features will turn out from the relativistic Newton's second law in the following sections.

3 Determining the Planck's constant

In this section we discuss how to determine the Planck's constant that emerges in the preceding section.

In 1900, M. Planck assumed that the energy of a harmornic oscilator can take on only discrete values which are integral multiples of $h\nu$, where ν is the vibration frequency and h is a fundamental constant, now either h or $\hbar = h/2\pi$ is called as Planck's constant. The Planck's constant next made its appearance in 1905, when Einstein used it to explain the photoelectric effect, he assumed that the energy in an electromagnetic wave of frequency ω is in the form of discrete quanta (photons) each of which has an energy $\hbar\omega$ in accordance with Planck's assumption. From then, it has been recognized that the

Planck's constant plays a key role in the quantum mechanics.

According to the previous section, no mater how to move or when to move in the Minkowski's space, the motion of a particle is governed by a potential function Φ as

$$mu_\mu + qA_\mu = \partial_\mu \Phi \tag{27}$$

For applying Eq.(27) to specific applications,without loss of generality, we set $\Phi = -i\kappa\psi$, then Eq.(27) is rewritten as

$$(mu_\mu + qA_\mu)\psi = -i\kappa\partial_\mu\psi \tag{28}$$

the coefficient κ is subject to the interpretation of ψ.

There are three mathematical properties of ψ worth recording here. First, if there is a path l_i joining initial point x_0 to final point x, then

$$\psi_i = e^{\frac{i}{\kappa}\int_{x_0(l_i)}^{x}(mu_\mu + qA_\mu)dx_\mu} \tag{29}$$

Second, the integral of Eq.(29) is independent from the choice of path. Third, the superposition principle is valid for ψ_i, i.e., if there are N paths from x_0 to x, then

$$\psi = \sum_i^N \psi_i \tag{30}$$

$$m\overline{u_\mu} = \sum_i^N mu_\mu\psi_i / \sum_i^N \psi_i \tag{31}$$

$$(m\overline{u_\mu} + qA_\mu)\psi = -i\kappa\partial_\mu\psi \tag{32}$$

where $m\overline{u_\mu}$ is average momentum.

To gain further insight into physical meanings of this equations, we shall discuss two applications.

3.1 Two slit experiment

As shown in FIG.2, suppose that the electron gun emits a burst of electrons at x_0 at time $t = 0$, the electrons arrive at the point x on the screen at time t. There are two paths for the electron to go to the destination, according to our above statement, ψ is given by

$$\psi = e^{\frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu)dx_\mu} + e^{\frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu)dx_\mu} \tag{33}$$

where we use l_1 and l_2 to denote the paths $a + b$ and $c + d$ respectively. Multiplying Eq.(33) by its complex conjugate gives

$$\begin{aligned} W &= \psi(x)\psi^*(x) \\ &= 2 + e^{\frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu)dx_\mu - \frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu)dx_\mu} \\ &\quad + e^{\frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu)dx_\mu - \frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu)dx_\mu} \\ &= 2 + 2\cos[\frac{1}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu)dx_\mu \\ &\quad -\frac{1}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu)d_\mu] \\ &= 2 + 2\cos[\frac{p}{\kappa}(l_1 - l_2)] \end{aligned} \tag{34}$$

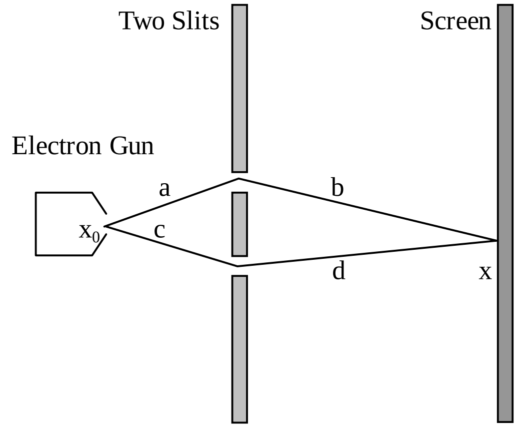

Figure 2: A diffraction experiment in which electron beam from the gun through the two slits to form a diffractin pattern at the screen.

where p is the momentum of the electron. We find a typical interference pattern with constructive interference when $l_1 - l_2$ is an integral multiple of κ/p, and destructive interference when it is a half integral multiple. This kind of experiments has been done a long time age, no mater what kind of particle, the comparison of the experiments to Eq.(34) leads to two consequences: (1) the complex function ψ is found to be probability amplitude, i.e., $\psi(x)\psi^*(x)$ expresses the probability of finding a particle at location x in the Minkowski's space. (2) κ is the Planck's constant.

3.2 The Aharonov-Bohm effect

Let us consider the modification of the two slit experiment, as shown in FIG.3. Between the two slits there is located a tiny solenoid S, designed so that a magnetic field perpendicular to the plane of the figure can be produced in its interior. No magnetic field is allowed outside the solenoid, and the walls of the solenoid are such that no electron can penetrate to the interior. Like Eq.(33), the amplitude ψ is given by

$$\psi = e^{\frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu + qA_\mu)dx_\mu} + e^{\frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu + qA_\mu)dx_\mu} \tag{35}$$

and the probability is given by

$$
\begin{aligned}
W &= \psi(x)\psi^*(x) \\
&= 2 + e^{\frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu+qA_\mu)dx_\mu - \frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu+qA_\mu)dx_\mu} \\
&\quad + e^{\frac{i}{\kappa}\int_{x_0(l_2)}^{x}(mu_\mu+qA_\mu)dx_\mu - \frac{i}{\kappa}\int_{x_0(l_1)}^{x}(mu_\mu+qA_\mu)dx_\mu} \\
&= 2 + 2\cos[\frac{p}{\kappa}(l_1 - l_2) + \frac{1}{\kappa}\int_{x_0(l_1)}^{x} qA_\mu dx_\mu \\
&\quad - \frac{1}{\kappa}\int_{x_0(l_2)}^{x} qA_\mu dx_\mu] \\
&= 2 + 2\cos[\frac{p}{\kappa}(l_1 - l_2) + \frac{1}{\kappa}\oint_{(l_1+\overline{l_2})} qA_\mu dx_\mu] \\
&= 2 + 2\cos[\frac{p}{\kappa}(l_1 - l_2) + \frac{q\phi}{\kappa}] \qquad (36)
\end{aligned}
$$

where $\overline{l_2}$ denotes the inverse path to the path l_2, ϕ is the magnetic flux that passes through the surface between the paths l_1 and $\overline{l_2}$, and it is just the flux inside the solenoid.

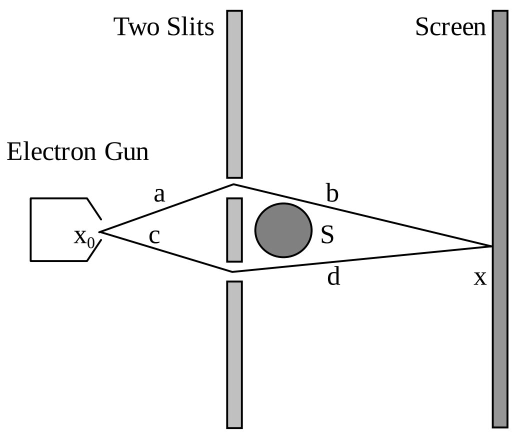

Figure 3: A diffraction experiment with adding a solenoid.

Now, constructive (or destructive) interference occurs when

$$\frac{p}{\kappa}(l_1 - l_2) + \frac{q\phi}{\kappa} = 2\pi n \quad (or \quad n + \frac{1}{2}) \qquad (37)$$

where n is an integer. When κ takes the value of the Planck's constant, we know that this effect is just the Aharonov-Bohm effect which was shown experimentally in 1960.

4 The directions of forces

In this section we shall correct a mistake about the Coulomb's force and the gravitational force in physical education, which cluttered the path from classical mechanics to quantum mechanics. We also shall discuss the Maxwell's equations in detail .

In the world, almost every young people was educated to know that the Coulomb's force and gravitational force act along the line joining a couple of particles, but these statements are incorrect in the relativity theory.

In relativity theory, the 4-vector velocity u of a particle has components u_μ, the magnitude of the 4-vector velocity u is given by

$$|u| = \sqrt{u_\mu u_\mu} = \sqrt{-c^2} = ic \qquad (38)$$

The above equation is valid so that any force can never change u in the magnitude but can change u in the direction. We therefore conclude that the Coulomb's force and gravitational force on a particle always act in the direction orthogonal to the 4-vector velocity of the particle in the 4-dimensional space-time, rather than along the line joining a couple of particles. Alternatively, any 4-vector force f satisfy the following perpendicular or orthogonal relation

$$u_\mu f_\mu = u_\mu m \frac{du_\mu}{d\tau} = \frac{m}{2}\frac{d(u_\mu u_\mu)}{d\tau} = 0 \qquad (39)$$

This simple inference clearly tells us that the forces are not centripetal or centrifugal forces, even if in 3-dimensional space, this character provides a internal reason for accounting for the quantum behavior of particle or chaos. Thus the derivations in the preceding section become reasonable.

In present paper, Eq.(39) has been elevated to an essential requirement for definition of force, which brings out many new aspects for the Coulomb's force and gravitational force.

4.1 The Coulomb's force and Lorentz force

We assume that the Coulomb's law remains valid only for two particles both at rest in 3-dimensional space. Suppose there are two charged particle q and q' locating at positions x and x' in the Cartesian coordinate system S and moving at 4-vector velocities u and u' respectively, as shown in FIG.4, where we use X to denote $x - x'$. The Coulomb's force f acting on particle q is perpendicular (orthogonal) to the velocity direction of q, as illustrated in FIG.4, like a centripetal force, the force f should make an attempt to rotate itself about its path center, the center may locate at the front or back of the particle q', so the force f should lie in the plane of u' and X, then

$$f = Au' + BX \qquad (40)$$

Where A and B are unknown coefficients, the possibility of this expansion will be further clear in the next subsection in where the expansion is not an assumption [see Eq.(61)]. Using the relation $f \perp u$, we get

$$u \cdot f = A(u \cdot u') + B(u \cdot X) = 0 \tag{41}$$

we rewrite Eq.(40) as

$$f = \frac{A}{u \cdot X}[(u \cdot X)u' - (u \cdot u')X] \tag{42}$$

It follows from the direction of Eq.(42) that the unit vector of the Coulomb's force direction is given by

$$\widehat{f} = \frac{1}{c^2 r}[(u \cdot X)u' - (u \cdot u')X] \tag{43}$$

because

$$\begin{aligned} \widehat{f} &= \frac{1}{c^2 r}[(u \cdot X)u' - (u \cdot u')X] \\ &= \frac{1}{c^2 r}[(u \cdot R)u' - (u \cdot u')R] \\ &= -[(\widehat{u} \cdot \widehat{R})\widehat{u}' - (\widehat{u} \cdot \widehat{u}')\widehat{R}] \\ &= -\widehat{u}' \cosh\alpha + \widehat{R} \sinh\alpha \end{aligned} \tag{44}$$

$$|\widehat{f}| = 1 \tag{45}$$

Where α refers to the angle between u and R, $R \perp u'$, $r = |R|$, $\widehat{u} = u/ic$, $\widehat{u}' = u'/ic$, $\widehat{R} = R/r$. Suppose that the magnitude of the force f has the classical form

$$|f| = k\frac{qq'}{r^2} \tag{46}$$

Combination of Eq.(46) with (43), we obtain a modified Coulomb's force

$$\begin{aligned} f &= \frac{kqq'}{c^2 r^3}[(u \cdot X)u' - (u \cdot u')X] \\ &= \frac{kqq'}{c^2 r^3}[(u \cdot R)u' - (u \cdot u')R] \end{aligned} \tag{47}$$

This force is in the form of the Lorentz force for the two particles, relating with the Ampere's law and Biot-Savart-Laplace law.

It is follows from Eq.(47) that the force can be rewritten in terms of 4-vector components as

$$f_\mu = qF_{\mu\nu}u_\nu \tag{48}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{49}$$

$$A_\mu = \frac{kq'}{c^2}\frac{u'_\mu}{r} \tag{50}$$

Where we have used the relations

$$\partial_\mu \left(\frac{1}{r}\right) = -\frac{R_\mu}{r^3} \tag{51}$$

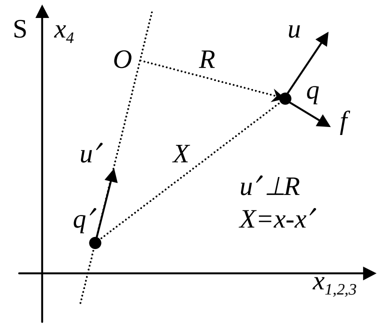

Figure 4: The Coulomb's force acting on q is perpendicular to the 4-vector velocity u of q, and lies in the plane of u' and X with the retardation with respect to q'.

4.2 The Lorentz gauge condition

From Eq.(50), because of $u' \perp R$, we have

$$\partial_\mu A_\mu = \frac{kq'u'_\mu}{c^2}\partial_\mu \left(\frac{1}{r}\right) = -\frac{kq'u'_\mu}{c^2}\left(\frac{R_\mu}{r^3}\right) = 0 \tag{52}$$

It is known as the Lorentz gauge condition.

4.3 The Maxwell's equations

To note that R has three degrees of freedom on the condition $R \perp u'$, so we have

$$\partial_\mu R_\mu = 3 \tag{53}$$

$$\partial_\mu \partial_\mu \left(\frac{1}{r}\right) = -4\pi\delta(R) \tag{54}$$

From Eq.(49), we have

$$\begin{aligned} \partial_\nu F_{\mu\nu} &= \partial_\nu\partial_\mu A_\nu - \partial_\nu\partial_\nu A_\mu = -\partial_\nu\partial_\nu A_\mu \\ &= -\frac{kq'u'_\mu}{c^2}\partial_\nu\partial_\nu\left(\frac{1}{r}\right) = \frac{kq'u'_\mu}{c^2}4\pi\delta(R) \\ &= \mu_0 J'_\nu \end{aligned} \tag{55}$$

where we define $J'_\nu = q'u'_\nu\delta(R)$. From Eq.(49), by exchanging the Indices and taking the summation of them, we have

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0 \tag{56}$$

The Eq.(55) and (56) are known as the Maxwell's equations. For continuous media, they are valid as well as.

4.4 The Lienard-Wiechert potential

From the Maxwell's equations, we know there is a retardation time for action to propagate between the two particles, the retardation effect is measured by

$$r = c\Delta t = c\frac{\overline{q'O}}{ic} = c\frac{\widehat{u}' \cdot X}{ic} = \frac{u'_\nu (x'_\nu - x_\nu)}{c} \tag{57}$$

as illustrated in FIG.4. Then

$$A_\mu = \frac{kq'}{c^2}\frac{u'_\mu}{r} = \frac{kq'}{c}\frac{u'_\mu}{u'_\nu (x'_\nu - x_\nu)} \tag{58}$$

Obviously, Eq.(58) is known as the Lienard-Wiechert potential for a moving particle.

The above formalism clearly shows that the Maxwell's equations can be derived from the classical Coulomb's force and the perpendicular relation of force and velocity. In other words, the perpendicular relation is hidden in the Maxwell's equation. Specially, Eq.(42) directly accounts for the geometrical meanings of curl of vector potential, the curl contains the perpendicular relation. Since the perpendicular relation of force and velocity is one of consequences from the relativistic Newton's second law, it also is one of the features from the particle invariance.

4.5 Gravitational force

The above formalism has a significance on guiding how to develope the gravity theory. In analogy with the modified Coulomb's force of Eq.(47), we directly suggest a modified universal gravitational force as

$$\begin{aligned} f &= -\frac{Gmm'}{c^2 r^3}[(u \cdot X)u' - (u \cdot u')X] \\ &= -\frac{Gmm'}{c^2 r^3}[(u \cdot R)u' - (u \cdot u')R] \end{aligned} \tag{59}$$

for a couple of particles with masses m and m' respectively.

Comparing with some incorrect statements about the Coulomb's force and gravitational force in most textbooks, the perpendicular (orthogonal) relation of force and velocity was called direction adaptation nature of force in the author's previous paper[11].

4.6 The Magnet-like components of the gravitational force

We emphasize that the perpendicular relation of force and velocity must be valid if the gravitational force can be defined as a force. It follows from Eq.(59) that we can predict that there are gravitational radiation and magnet-like components for the gravitational force. Particularly, the magnet-like components will act as a key role in the geophysics and atmosphere physics.

If we have not any knowledge but know there exists the classical universal gravitation $\mathbf{f}$ between two particles m and m', what form will take the 4-vector gravitational force f ? Suppose that m' is at rest at the origin, using $u = (\mathbf{u}, u_4)$, $u' = (0, 0, 0, ic)$ and $u \cdot f = 0$, we have

$$f_4 = \frac{u_4 f_4}{u_4} = \frac{u \cdot f - \mathbf{u} \cdot \mathbf{f}}{u_4} = -\frac{\mathbf{u} \cdot \mathbf{f}}{u_4} \tag{60}$$

$$\begin{aligned} f &= (\mathbf{f}, f_4) = \mathbf{f} + f_4\frac{u'}{ic} = \mathbf{f} - (\frac{\mathbf{u} \cdot \mathbf{f}}{u_4})\frac{u'}{ic} \\ &= \frac{1}{icu_4}[icu_4\mathbf{f} - (\mathbf{u} \cdot \mathbf{f})u'] \\ &= \frac{1}{icu_4}[(u' \cdot u)\mathbf{f} - (\mathbf{u} \cdot \mathbf{f})u'] \\ &= \frac{|\mathbf{f}|}{icu_4|\mathbf{R}|}[(u' \cdot u)\mathbf{R} - (\mathbf{u} \cdot \mathbf{R})u'] \\ &= \frac{|\mathbf{f}|}{icu_4|\mathbf{R}|}[(u' \cdot u)R - (u \cdot R)u'] \end{aligned} \tag{61}$$

where $R \perp u'$, $R = (\mathbf{R}, 0)$. If we rotate our frame of reference to make m' not to be at rest, Eq.(61) will still valid because of covariance. Then we find the 4-vector gravitational force goes back to the form of Eq.(59), like the Lorentz force, having the magnet-like components.

It is noted that the perpendicular relation of force and velocity is valid for any force: strong, electromagnetic, weak and gravitational interactions, therefore there are many new aspects remaining for physics to explore.

5 Interactions between particles

Since quarks have never been observed, our speculation leads us to propose a better model to organize known data. For this challenging purpose, in the present paper, we introduce a fictitious elementary particle, given a name "Dollon" for our convenience, to resemble other particles such as fermions, mesons or classical particles.

5.1 Basic force

Consider a Dollon moving in the Minkowski's space $(x_1, x_2, x_3, x_4 = ict)$ with 4-vector velocity $u = (\mathbf{u}, u_4)$, then the motion of the particle Dollon satisfies the magnitude formula of 4-vector velocity of particle

$$u_\mu u_\mu = -c^2 \tag{62}$$

Differentiating the above equation with respect to proper time interval $d\tau$ gives

$$\frac{d\mathbf{u}}{d\tau} = \mathbf{f} \qquad \frac{du_4}{d\tau} = -\frac{\mathbf{u} \cdot \mathbf{f}}{u_4} \tag{63}$$

where the result has been written in the two parts by defining a 3-dimension spatial vector $\mathbf{f}$. Defining a 4-vector

$$f = (\mathbf{f}, -\frac{\mathbf{u}\cdot\mathbf{f}}{u_4}) \qquad (64)$$

then from Eq.(62) we have readily

$$\frac{du}{d\tau} = f \qquad u\cdot f = u_\mu f_\mu = 0 \qquad (65)$$

It means that u and f are orthogonal with each other.

Consider two particles "Bob" and "Alice" located at x and x' in the 4-dimensional space respectively, they composed of many Dollons, the number of Dollons in Alics is M, and in Bob is m, when "Bob" and "Alice" move with 4-vector velocities u and u' respectively, following Eq.(63), they can be assigned two sets of motion equations as

$$Bob: m\frac{d\mathbf{u}}{d\tau} = m\mathbf{f} \qquad m\frac{du_4}{d\tau} = -m\frac{\mathbf{u}\cdot\mathbf{f}}{u_4} \qquad (66)$$

$$Alice: \quad M\frac{d\mathbf{u}'}{d\tau'} = M\mathbf{f}' \qquad M\frac{du_4'}{d\tau'} = -M\frac{\mathbf{u}'\cdot\mathbf{f}'}{u_4'} \qquad (67)$$

Now we have a question: what is the interaction between Bob and Alice? Obviously, the form of Eq.(66) seems to be the relativistic Newton second law for Bob, $\mathbf{f}$ seems to be a 3-vector force, $\mathbf{u}\cdot\mathbf{f}$ seems to be the rate at which the force does work on Bob. For seeking further answer, we need to recall the Newton's first law of motion, the law remains valid in the relativistic theory and reads

First Law: An object at rest will remain at rest and an object in motion will continue to move in a straight line at constant speed forever unless some net external force acts to change this motion.

If the object is a composite system composed of many Dollons, then we understand the First Law with two consequences.

Consequence 1: Let S denote the number of Dollons in a composite system, the velocity of the center of the system is defined as

$$u_c = \frac{1}{S}\sum_i^S u^{(i)} \qquad (68)$$

where $u^{(i)}$ is the 4-vector velocity of the ith Dollon. The Fist Law only means that the center of the system remains at rest or in motion, i.e., the rotation about its center is allowed.

Consequence 2: The total number of Dollons in the system must be unchanged, i.e., the conservation of Dollon number, otherwise any creation or annihilation of Dollon will lead to a shift of the center of the system.

Now we go back to consider the whole system composed of Bob and Alics, without loss of generality, suppose that the center is at rest at the origin of the frame of reference, then the center has a 4-vector velocity $u_c = (0, 0, 0, u_{4c})$, the "at rest" refers to being at rest in 3-dimensional space. From Eq.(65), the quantity f must be orthogonal with the 4-vector velocity u of Bob, likewise for Alice, we have

$$Bob \quad : \qquad u\cdot f = u_\mu f_\mu = 0 \qquad (69)$$

$$Alice \quad : \qquad u'\cdot f' = u'_\mu f'_\mu = 0 \qquad (70)$$

They set up a rule for the interaction between Bob and Alice in the composite system. We specially choose to study the interaction which happens at a such moment that their positions are just joining by a 4-vector R, the 4-vector R is the position vector of Bob with respect with Alice, $R = x - x'$, and R is orthogonal to u and u' simultaneously.

$$Bob \quad : \qquad u\cdot R = 0 \qquad (71)$$

$$Alice \quad : \qquad u'\cdot R = 0 \qquad (72)$$

Then from Eq.(69) and Eq.(70), we get

$$Bob \quad : \qquad f \propto R \qquad (73)$$

$$Alice \quad : \qquad f' \propto R \qquad (74)$$

For Bob, using notation $R = (\mathbf{r}, R_4)$, $r = |\mathbf{r}|$, multiplying Eq.(66) by $\mathbf{r}$, we have

$$\mathbf{r}\times(m\frac{d\mathbf{u}}{d\tau}) = m\frac{d(\mathbf{r}\times\mathbf{u})}{d\tau} = m\mathbf{r}\times\mathbf{f} = \mathbf{0} \qquad (75)$$

It means

$$\mathbf{r}\times\mathbf{u} = h = const. \qquad (76)$$

where h is an integral constant. Likewise for Alice. From Eq.(73) we can expand $\mathbf{f}/u_4$ in a Taylor series in $1/r$, this gives

$$\frac{\mathbf{f}}{u_4} = \frac{\mathbf{r}}{r}(b_0 + b_1\frac{1}{r} + b_2\frac{1}{r^2} + b_3\frac{1}{r^3} + ...) \qquad (77)$$

According to the First Law, when Bob and Alice is separated by a infinite distance, they can be seen as two isolated particles, the interaction between them must completely vanish, so we have $b_0 = 0$ in Eq.(77). From Eq.(66) we obtain

$$\begin{aligned} u_4 &= \int(-\frac{\mathbf{u}\cdot\mathbf{f}}{u_4})d\tau = \int\frac{|\mathbf{f}|}{u_4}dr \\ &= \varepsilon - b_1\ln r + b_2\frac{1}{r} + b_3\frac{1}{2r^2} + ... \end{aligned} \qquad (78)$$

where ε is an integral constant. For the same reason like b_0,we find that b_1 must be zero. Now consider Eq.(76), it means that Bob moves around Alice, when $h \to 0$, Bob may access Alice as close as possible at perihelion point, at the perihelion point we find

$$\begin{aligned} h &= \mathbf{r} \times \mathbf{u} = r u_\varphi|_{perihelionpoint} = r\sqrt{-c^2 - u_4^2} \\ &= \sqrt{r^2[-c^2 - (\varepsilon + b_2\frac{1}{r} + b_3\frac{1}{2r^2} + ...)^2]} \end{aligned} \tag{79}$$

For h preserves an constant when $r \to 0$, then we need all coefficients b_i to be zero but except b_2 in the above equation. Therefore we obtain

$$\frac{\mathbf{f}}{u_4} = b\frac{\mathbf{r}}{r^3} \tag{80}$$

$$u_4 = \varepsilon + b\frac{1}{r} \tag{81}$$

where the subscript of b_2 have been dropped. Likewise for Alice, we have

$$\frac{\mathbf{f}'}{u_4'} = a\frac{\mathbf{r}}{r^3} \tag{82}$$

$$u_4' = \varepsilon' + a\frac{1}{r} \tag{83}$$

where a is a coefficient.

Differentiating Eq.(68) with respect to time interval dt gives the center velocity as

$$\mathbf{u}_c = 0 = (\frac{icm}{u_4}\frac{d\mathbf{u}}{d\tau} + \frac{icM}{u_4'}\frac{d\mathbf{u}'}{d\tau'})/(m+M) \tag{84}$$

where we have used $u_4 = dx_4/d\tau = icdt/d\tau$, $u_4' = icdt/d\tau'$, $x_4 = ict + x_4(0)$ and $x_4' = ict + x_4'(0)$. Substituting Eq.(80) and Eq.(82) into Eq.(84), we get

$$icm\frac{\mathbf{f}}{u_4} + icM\frac{\mathbf{f}'}{u_4'} = b\frac{icm\mathbf{r}}{r^3} + a\frac{icM\mathbf{r}}{r^3} = 0 \tag{85}$$

This equation leads to

$$\frac{b}{M} = -\frac{a}{m} = K \tag{86}$$

where K is an constant. Then Eq.(66) and Eq.(67) may be rewritten as

$$Bob: \quad m\frac{d\mathbf{u}}{d\tau} = K\frac{mM\mathbf{r}}{r^3} \tag{87}$$

$$Alice: \quad M\frac{d\mathbf{u}'}{d\tau'} = -K\frac{mM\mathbf{r}}{r^3} \tag{88}$$

If K takes a negative constant, then, the above equations show that Bob is attracted by Alice with the Newton's universal gravitation force. But we do not want to make this conclusion at once, because there are still a few problems among them.

5.2 The Coulomb's force

From the above subsection, now we can manifestly interpret the quantity f to be the 4-vector force exerted on a Dollon of Bob. It is a natural idea to think of that Dollon has two kinds of charges: positive and negative. If Bob and Alice are separated by a far distance, and f is the force acting on a positive Dollon in Bob, then $-f$ is the force acting on a negative Dollon in Bob, it follows from Eq.(63) that the motion of the ith Dollon is governed by

$$\frac{du^{(i)}}{d\tau^{(i)}} = f^{(i)} \quad or \quad \frac{du^{(i)}}{dt} = \frac{f^{(i)}}{u_4^{(i)}} \tag{89}$$

where $d\tau^{(i)}$, $u^{(i)}$ and $f^{(i)}$ denote the proper time interval, 4-vector velocity and 4-vector force of the ith Dollon, respectively. Taking sum over all Dollons in Bob, we get

$$\sum_{i=1}^{m}\frac{du^{(i)}}{dt} = \frac{d}{dt}[\sum_{i=1}^{m}u^{(i)}] = \frac{d(mu_c)}{dt} \tag{90}$$

$$\sum_{i=1}^{m}\frac{f^{(i)}}{u_4^{(i)}} \simeq q\frac{f_c}{u_{c4}} \tag{91}$$

where u_c is the 4-vector velocity of the center of Bob, u_{c4} denotes its 4th component, q denotes the net charges of Bob, f_c denotes the 4-vector force of a virtual Dollon located at the center of Bob. Combining Eq.(90) and Eq.(91) with Eq.(89), we obtain

$$\frac{d(mu_c)}{dt} = q\frac{f_c}{u_{c4}} \quad or \quad \frac{d(mu_c)}{d\tau_c} = qf_c \tag{92}$$

Like that in the above subsection, the First Law must be valid for the composite system of Bob and Alice, in other words, when they are separated from a infinite distance they are isolated, while when they go to nearest point they do not touch each other, these requirements lead to

$$Bob \quad : \quad \frac{\mathbf{f}_c}{u_{c4}} = b\frac{\mathbf{r}}{r^3} \qquad u_{c4} = \varepsilon + b\frac{1}{r} \tag{93}$$

$$Alice \quad : \quad \frac{\mathbf{f}_c'}{u_{c4}'} = a\frac{\mathbf{r}}{r^3} \qquad u_{c4}' = \varepsilon' + a\frac{1}{r} \tag{94}$$

where $\varepsilon, \varepsilon'$, b and a are coefficients. Without loss of generality, we have

$$\mathbf{u}_c = 0 = (\frac{ic}{u_{c4}}\frac{d(m\mathbf{u}_c)}{d\tau_c} + \frac{ic}{u_{c4}'}\frac{d(M\mathbf{u}_c')}{d\tau_c'})/(m+M) \tag{95}$$

Substituting Eq.(93) and Eq.(94) into Eq.(95), we get

$$icm\frac{\mathbf{f}_c}{u_{c4}} + icM\frac{\mathbf{f}'_c}{u'_{c4}} = b\frac{icq\mathbf{r}}{r^3} + a\frac{icq'\mathbf{r}}{r^3} = 0 \tag{96}$$

where q' denotes the net charges of Alice. This equation leads to

$$\frac{b}{q} = -\frac{a}{q'} = k \tag{97}$$

where k is a constant. Then the motions of Bob and Alice are governed by

$$Bob: \quad m\frac{d\mathbf{u}}{d\tau} = k\frac{qq'\mathbf{r}}{r^3} \tag{98}$$

$$Alice: \quad M\frac{d\mathbf{u}'}{d\tau'} = -k\frac{qq'\mathbf{r}}{r^3} \tag{99}$$

The 4th components corresponding to the above equations express the energy change rates of Bob and Alice, they are not independent equations, this explanation can be found in the relativity theory.

The Eq.(98) and Eq.(99) are known as the Coulomb's forces.

5.3 Gravitational force

If Bob and Alice are two atoms with neutral net charges, then the Coulomb's force between them vanish off. But, precisely, this is not true, the inspection of Eq.(91) tells us that the net interaction between them still remains when the atoms are considered as composite systems.

In this paper, planet, stone, molecule, atom and nucleus are all regarded as the composite systems composed of Dollon, the constituents of the composite systems move about their centers. If Bob and Alice are two planets with neutral net charges, then it is reasonable to assume that the net force acting on Bob is proportional to the number of Dollons in Bob, Eq.(91) reads

$$\sum_{i=1}^{m}\frac{f(i)}{u_4(i)} = gm\frac{f_c}{u_{c4}} \tag{100}$$

where g is a proportional coefficient. Then the motion of Bob is given by

$$\frac{d(mu_c)}{dt} = gm\frac{f_c}{u_{c4}} \quad or \quad \frac{d(mu_c)}{d\tau_c} = gmf_c \tag{101}$$

In analogy with the above subsections, we may obtain the motion equations of Bob and Alice, they are governed by

$$Bob: \quad m\frac{d\mathbf{u}}{d\tau} = -G\frac{mM\mathbf{r}}{r^3} \tag{102}$$

$$Alice: \quad M\frac{d\mathbf{u}'}{d\tau'} = G\frac{mM\mathbf{r}}{r^3} \tag{103}$$

The m and M has been identified or defined as the masses by employing Dollon mass as a unit when we count the Dollon numbers in Bob or Alice. The Eq.(102) and Eq.(103) are known as the Newton's universal gravitational forces. Why is the net force of Bob attractive ? This may be explained as that electrons with light mass move always around massive nuclei, the attraction is a little bigger than the repulsion between two atoms separated by a far distance. In this formulation, the gravitational force possesses statistic meanings.

5.4 Nuclear force

Now consider that Bob and Alice are two nucleons composed of Dollons. If Bob and Alice go closely in a distance comparable with the sizes of them, then it is clear that Eq.(91) turns to be inadequate, their polarization can provide a strong interaction, while the effect of net charges between their centers becomes to be trivial. Therefore, the strong nuclear force is charge-independent, it only comes into play when the nucleons are very close together, and it drops rapidly to zero for larger distance, we know from experiments that the sensitive distance is about $10^{-15}m$.

As mentioned above, the ith Dollon in Bob is governed by

$$\frac{d\mathbf{u}^{(i)}}{d\tau^{(i)}} = \mathbf{f}^{(i)} \qquad \frac{du_4^{(i)}}{d\tau^{(i)}} = -\frac{\mathbf{u}^{(i)}\cdot\mathbf{f}^{(i)}}{u_4^{(i)}} \tag{104}$$

Then the motion of Bob is given by

$$\frac{d(m\mathbf{u}_c)}{dt} = \sum_{i=1}^{m}\frac{\mathbf{f}^{(i)}}{u_4^{(i)}} \tag{105}$$

$$\frac{d(mu_{c4})}{dt} = -\sum_{i=1}^{m}\frac{\mathbf{u}^{(i)}\cdot\mathbf{f}^{(i)}}{[u_4^{(i)}]^2} \tag{106}$$

where

$$mu_c = \sum_{i=1}^{m}u^{(i)} \tag{107}$$

But u_c is not the *relativistic* velocity of the center of Bob, the *relativistic* velocity of the center of Bob is defined by using its center proper time, so that $u_{center} = dx_{center}/d\tau_{center}$, we may establish their relation by introducing a correcting factor λ so that $u_c = \lambda u_{center}$, i.e.

$$\sum_{i=1}^{m}u^{(i)} = mu_c = \lambda u_{center} \tag{108}$$

In the following we drop the subscript "center" when without confusion, then above equations can be rewritten as

$$\frac{d(m\lambda \mathbf{u})}{dt} = \sum_{i=1}^{m} \frac{\mathbf{f}^{(i)}}{u_4^{(i)}} \tag{109}$$

$$\frac{d(m\lambda u_4)}{dt} = -\sum_{i=1}^{m} \frac{\mathbf{u}^{(i)} \cdot \mathbf{f}^{(i)}}{[u_4^{(i)}]^2} \tag{110}$$

It is noted that the right side of Eq.(110) is the rate at which the forces do works on Bob, then the quantity $-m\lambda u_4$ in the left side should be the energy, we define the energy by

$$E = -ic\lambda m u_{c4} = m_r \lambda c^2 \tag{111}$$

$$m_r = \frac{m}{\sqrt{1 - v^2/c^2}} \tag{112}$$

where $u_{c4} = ic/\sqrt{1 - v^2/c^2}$, v is the classical speed of the center of Bob, m_r is the relativistic mass, while m is the rest mass. Eq.(111) has a little difference from the Einstein's mass-energy relationship. Our energy formula contains a factor λ that represents the internal motion of Dollons in Bob, obviously, $\lambda \geq 1$, this can be seen clearly from Eq.(108), in other words, even if the center is at rest, the internal constituents can still have relativistic energies.

In dealing with nuclear reaction, in many textbooks, the mass defect is understood as the decrease in total relativistic mass, even if all nuclei seem to be at rest before or after the nuclear reaction. Further, it was said that when the nucleus was formed the excess mass was radiated away in the form of energy. It is a long time we are puzzled by these statements. Now the reasons are clear, no relativistic masses change but λ changes in these cases, in other words, the internal energy of particle has changed.

Consider that a hadron possesses net charge q, we can naturally image that the charge distributes in several parts inside the hadron, assuming three parts, each part has net charge denoted by I_q, B_q, and S_q respectively, then

$$q = I_q + B_q + S_q \tag{113}$$

Comparing with the Gell-Mann-Nishijima relation

$$Q = I_3 + \frac{B + S}{2} \tag{114}$$

we can understand the conservations of isospin I_3, baryon number B and strangeness number S with four remarks: (1) the three parts inside the hadron are insulated from one another, no charge transports from one to another. (2) during collision of hadrons, only the identical parts impact or touch each other, with exchanging the net charges. (3) the mass of the hadron seems to depend primarily on the masses of the parts inside the hadron, weakly on the net charges of the parts. (4) if we assign the quantum states of quarks u, s and d to the three parts, the quark model seems to be improved in a manner that we can avoid the fractional charges of the quarks.

5.5 Determining the 4-vector R

In the preceding subsections, we have mentioned that the interaction between Bob and Alice we studied happens at such moment that their relative position in the Minkowski's space is denoted by a 4-vector $R = (\mathbf{R}, R_4) = (\mathbf{r}, ic\triangle t)$, R satisfies the orthogonal relation

$$u \cdot R = u' \cdot R = 0 \tag{115}$$

The purpose of choosing this moment is to meet convenience that R parallels to f and f' simultaneously, because

$$Bob \quad : \quad u \cdot f = u_\mu f_\mu = 0 \tag{116}$$

$$Alice \quad : \quad u' \cdot f' = u'_\mu f'_\mu = 0 \tag{117}$$

See Eq.(69)-(74). Eq.(115) can be rewritten in the form of inner product of two vectors as

$$|u| \cdot |R| \cosh(u, R) = |u'| \cdot |R| \cosh(u', R) = 0 \tag{118}$$

This leads to a solution given by

$$|R| = 0 \tag{119}$$

This again leads to two solutions given by

$$|\mathbf{r}| = r = c\triangle t \qquad r = -c\triangle t \tag{120}$$

The first solution expresses that the force acting on Bob is retarded by time $\triangle t = r/c$, the second one expresses that the action is preceded. Our choice is the first one which gives an effect that follows the cause. We know, this retarded time is just the time needed for the propagation of interaction from Alice to Bob, the propagation speed is c, no mater what kind of interaction.

6 The Minkowski's space

In preceding sections, we have realized that the relativistic Newton's second law and forces can be derived from the Newton's first law and the magnitude formula of 4-vector velocity of particle. The formula is given by

$$u_\mu u_\mu = -c^2 \tag{121}$$

in a Minkowski's space. It is noted that all particles satisfy the above equation, it then is regarded as the origin of the particle invariance. We wonder, what is the Minkowski's space. In this section we shall discuss the Minkowski's space, for this purpose we need to establish a standard method for describing the motion of particle in space-time. Our construction follows five steps.

6.1 First Step: In those lazy days

Suppose Alice is a pretty girl famous for his fast running records, we state some his records here in a story (in imagination).

(1) Jan., 1, 2001, 10:00 am, sportsground at BUAA, Beijing.

In a time interval $\Delta t = 10s$ Alice run a straight line distance $\Delta l_1 = 100m$ at a constant speed $v_1 = 10m/s$. This data can be given in physical terms by

$$\Delta l_1 = v_1 \Delta t \tag{122}$$

It can be rewritten either as

$$\Delta x_1^2 + \Delta y_1^2 = (v_1 \Delta t)^2 \tag{123}$$

or as

$$\Delta x_1^2 + \Delta y_1^2 - (v_1 \Delta t)^2 = 0 \tag{124}$$

where x and y denote the coordinate system fixed at the sportsground. By defining a imaginary quantity $\Delta w_1 = iv_1 \Delta t$, the data is given by

$$\Delta x_1^2 + \Delta y_1^2 + \Delta w_1^2 = 0 \tag{125}$$

We may appreciate the simplicity and beauty of its form.

It is also our favorite manner to mark the running process in a graph with three mutually perpendicular axes x, y and $w = iv_1 t$. The distance from the start point to the final point in this coordinate system equals to zero because of Eq.(125). This graph we call "20010101 Graph".

(2) Jan., 2, 2001, 10:00 am, sportsground at BUAA, Beijing.

In a time interval $\Delta t = 10s$ Alice run a straight line distance $\Delta l_2 = 90m$ at a constant speed $v_2 = 9m/s$. This data is given in physical terms by

$$\Delta x_2^2 + \Delta y_2^2 + \Delta w_2^2 = 0 \qquad \Delta w_2 = iv_2 \Delta t \tag{126}$$

We directly mark today running process in yesterday 20010101 Graph, we are lazy to draw a new graph.

(3) Jan., 3, 2001, 10:00 am, sportsground at BUAA, Beijing.

In a time interval $\Delta t = 10s$ Alice run a straight line distance $\Delta l_3 = 95m$ at a constant speed $v_3 = 9.5m/s$. We also directly mark the running process in the 20010101 Graph.

Bob is also a good runner, in a time interval $\Delta t = 10s$ Bob run a straight line distance $\Delta l_b = 105m$ at a constant speed $v_b = 10.5m/s$. We also directly mark the running process in the 20010101 Graph.

In fact, their running data all is marked in the 20010101 Graph.

6.2 Second Step: Establishing a temporary standard frame

In those lazy days, we only used the 20010101 Graph to record the running data, it actually become a temporary standard frame, all motions can be marked or calculated in the Graph, it is much connivent for describing any movement.

We note that $w = iv_1 t$, the w axis in the Graph had involved the speed v_1 created by Alice on Jan 1, 2001. Thus we find that the geometrical distance Δs_2 from the start point to the final point in the Graph on the second day for Alice is not equal to zero.

$$\Delta s_2^2 = \Delta x_2^2 + \Delta y_2^2 + \Delta w_1^2 \neq 0 \qquad \Delta w_1 = iv_1 \Delta t \tag{127}$$

It is clear after comparing with Eq.(126). So do for Bob, the distance Δs_b for Bob in the 20010101 Graph is given by

$$\begin{aligned} \Delta s_b^2 &= \Delta x_b^2 + \Delta y_b^2 + \Delta w_1^2 \\ &= \Delta x_b^2 + \Delta y_b^2 - v_1^2 \Delta t^2 \\ &= \Delta x_b^2 + \Delta y_b^2 - v_b^2 \Delta t^2 + v_b^2 \Delta t^2 - v_1^2 \Delta t^2 \\ &= v_b^2 \Delta t^2 - v_1^2 \Delta t^2 \\ &= -v_1^2 \Delta t^2 (1 - v_b^2/v_1^2) \end{aligned} \tag{128}$$

$$\Delta x_b^2 + \Delta y_b^2 - v_1^2 \Delta t^2 = -v_1^2 \Delta t^2 (1 - v_b^2/v_1^2) \tag{129}$$

Dividing the two sides of the above equation by $\Delta t^2(1 - v_b^2/v_1^2)$, we get

$$\left(\frac{\Delta x_b/\Delta t}{\sqrt{1 - v_b^2/v_1^2}}\right)^2 + \left(\frac{\Delta y_b/\Delta t}{\sqrt{1 - v_b^2/v_1^2}}\right)^2 + \left(\frac{iv_1}{\sqrt{1 - v_b^2/v_1^2}}\right)^2 = -v_1^2 \tag{130}$$

Defining modified velocity

$$u_x = \frac{v_x}{\sqrt{1 - v^2/v_1^2}} \tag{131}$$

$$u_y = \frac{v_y}{\sqrt{1 - v^2/v_1^2}} \tag{132}$$

$$u_w = \frac{iv_1}{\sqrt{1 - v^2/v_1^2}} \tag{133}$$

where we have dropped the subscript b for indicating Bob, then Eq.(130) is given by

$$u_x^2 + u_y^2 + u_w^2 = -v_1^2 \tag{134}$$

The modified velocity of Bob in the 20010101Graph is based on the Alice's best speed record v_1. In fact, Any one, any body or any particle, their modified velocity in the 20010101 Graph satisfies Eq.(134).

6.3 Third Step: Standard Graph based on the light speed

For some reasons, it has become a habit for us to use the 20010101 Graph to mark the all motions of any body. Some day, we find Chris runs faster than Alice, then we recognize that we need a permanent runner for establishing a standard graph. Now we have to face a new task: to look for a new hero.

It is said that the light, an element of the nature, is the fastest runner, whenever and whereever its speed is $3 \times 10^8 m/s$. We do not hesitate to use the light speed to replace Alice's speed, and setup a new frame called "Standard Graph", the Standard Graph contains four mutually perpendicular axes x, y, z and $w = ict$. From then, any motions can be described in the Standard Graph with the space-time (x, y, z, ict) or $(x_1, x_2, x_3, x_4 = ict)$. In analogy with Eq.(131)-(134), defining modified velocity

$$u_x = \frac{v_x}{\sqrt{1 - v^2/c^2}} \qquad u_y = \frac{v_y}{\sqrt{1 - v^2/c^2}} \tag{135}$$

$$u_z = \frac{v_z}{\sqrt{1 - v^2/c^2}} \qquad u_w = \frac{ic}{\sqrt{1 - v^2/c^2}} \tag{136}$$

we obtain

$$u_x^2 + u_y^2 + u_z^2 + u_w^2 = -c^2 \qquad u_\mu u_\mu = -c^2 \tag{137}$$

The Standard Graph is just the Minkowski's space, the 4-vector velocity $u = \{u_\mu\}$ is known as the relativistic velocity.

6.4 Fifth Step: Transformations

We immediately recognize that physics holds its validity only in the Standard Graph (involved with the light speed), rather than in the 20010101 Graph (involved with the Alice's speed), this situation may be explained by the facts that all physical quantities and their measurements were defined on facilities whose principles were based on the light directly or indirectly, for example, the "meter" and "second" were defined on the light speed directly, the fact is enough.

If we do not hope that one graph has advantage over than another, then the transformations between the Standard Graph and 20010101 Graph is given by

$$x = x_1, y = y_1, z = z_1, t = \frac{v_1}{c} t_1 \tag{138}$$

where the subscript 1 denotes in the 20010101 Graph. It means we need to redefine all physical quantities such as rod and clock in the 20010101 Graph, do not use the light.

7 Dynamics in the Hilbert space

A complete inner product space is called a Hilbert space. Our experience in the preceding sections tells us that it is an easy thing to put dynamics into the Hilbert space if we have an invariant quantity. The formalism of the interaction can be derived from some basic laws, it is strongly based on concrete instances.

8 Conclusions

It is important to recognize that physics must be invariant for the composite particles and their constituent particles, only one physical formalism exists for any particle, this requirement is called particle invariance.

Under the particle invariance, it is rather remarkable that the Klein-Gordon equation and the Dirac equation can be derived from the relativistic Newton's second law on different conditions respectively, thus only one formalism is necessary for particle, the relativistic Newton's second law is regarded as one which suitable for any kinds of particles.

It is pointed out that the gravitational force and Coulomb's force on a particle always act in the direction perpendicular to the 4-vector velocity of the particle in 4-dimensional space-time, rather than along the line joining a couple of particles. This inference is obviously supported from the fact that the magnitude of the 4-vector velocity is kept constant. The Maxwell's equations can be derived from the classical Coulomb's force and the magnitude formula of 4-vector velocity of particle.

Our speculation on the quarks model leads to introduce a new elementary particle called Dollon to resemble particles such as baryons, mesons and other composite particles, instead of quarks model, the Dollon model is better in organizing known data, specially in modelling interactions. It is found that the relativistic Newton's second law and various interactions can be derived from the Newton's first law and the magnitude formula of 4-vector velocity of particle.

The structure of the Minkowski's space is discussed in detail, it indicates that the magnitude formula of 4-vector velocity of particle is only a geometrical distance formula, so it is completely free from any particle property. Any dynamics or dynamical characteristics originated from the magnitude formula of 4-vector velocity of particle will completely preserve the particle invariance, i.e., the dynamics do not distinguish particle species. Thus the magnitude formula of 4-vector velocity of particle is regarded as the origin of the particle invariance.

Dynamics in the Hilbert space can be established in the same way.

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